Homomorphisms of relational systems and the corresponding groupoids

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We study relational systems with one binary relation. Quotient relational systems are introduced and some of their properties are characterized. Homomorphisms, strong mappings and cone preserving mappings are treated and the connections between them are considered. To every relational system is assigned a corresponding groupoid and the properties of these groupoids are investigated.

On associatively augmented nearlattices

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A *nearlattice* is a meet semilattice A in which every initial segment $A_p := \{x\colon x \leq p\}$ happens to be a join semilattice (hence, a lattice) with respect to the natural ordering of A. If all lattices A_p are distributive, the nearlattice itself is said to be *distributive.* It is known that a distributive nearlattice can be represented as a nearlattice of sets.

We call an algebra $(A, \wedge, \vee)$ of type (2,2) an *augmented nearlattice* if the following holds:

(i) $(A, \wedge)$ is a meet semilattice,

(ii) for every p, $x \vee y$ is the join of x and y whenever $x, y \in A_p$,

(iii) $x \leq y$ only if $x \vee y = y = y \vee x$ (the converse is alwaus true),

and say that it is *associatively augmented* if the operation $\vee$ is associative.

The class of all augmented nearlattices is a variety. We discuss some motivating examples and present a representation theorem for distributive associatively augmented nearlattices.

Eigenvalues of matrix products, generalized eigenvector centrality and applications

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Eigenvector centrality is a linear algebra based graph invariant used in various rating systems such as webpage ratings for search engines.

We propose a generalization of the eigenvector centrality invariant which can be used to define a rating system for bipartite graphs modeling time-critical processes.

We describe an example, the linear algebra connection and some results.

References

[1] M. E. J. Newman, *The mathematics of networks*, in The New Palgrave Encyclopedia of Economics, 2nd edition, L. E. Blume and S. N. Durlauf (eds.), Palgrave Macmillan, Basingstoke (2008).

De Morgan algebras and De Morgan quasirings

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De Morgan quasirings are connected to De Morgan algebras in the same way as Boolean rings are connected to Boolean algebras. We present a common axiom system for both De Morgan quasirings and De Morgan algebras and show how an interval of a De Morgan algebra (or De Morgan quasiring) can be viewed as a De Morgan algebra (or De Morgan quasiring, respectively).

References

[1] I. Chajda and G. Eigenthaler: De Morgan quasirings, Contributions to General Algebra 18 (Verlag J. Heyn, Klagenfurt 2008), 17-22.

[2] I. Chajda and G. Eigenthaler: Two constructions of De Morgan algebras and De Morgan quasirings, Discussiones Mathematicae – General Algebra and Applications, Vol. 29, No. 2 (2009), to appear.

Interaction structures

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An *interaction algebra* is a heterogeneous algebra $\langle A, S, T\rangle$ with sorts
A — algebra of interacting elements (agents);
S — the spatial structure;
T — time (usually discrete, may be nonsynchroneous)
together with interaction/communication rules. Examples of interaction algebras are:

- Wolfram's cell-automata ([1]), the game of Life ([2]), but also the game Pacman ([3]);
- simulation of emergence of common vocabulary ([4], [5]);
- structures extensively studied in connection with alife ([6], [7]).

References

[1] http://mathworld.wolfram.com/ElementaryCellularAutomaton.html

[2] http://www.bitstorm.org/gameoflife/

[3] http://www.pacmangame.net/

[4] Jaak Henno. Emergence of Names and Compositionality. Information Modelling and Knowledge Bases XVIII, IOS Press, Amserdam 2007, pp 80–100.

[5] JaakHenno. Emergence of Language: Hidden States and Local Encironments. Information Modelling and Knowledge Bases XIX, Hannu Jaakkola, Yasushi Kiyoki and Takahiro Tokuda (eds), IOS Press Amsterdam-Berlin-Oxford-Tokyo-Washington DC, pp 170–181.

[6] http://alife.org/

[7] http://www.santafe.edu/research/

$M(r,s)$-ideals of compact operators

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The subspace $\mathcal{K}(X,Y)$ of compact operators from a Banach space X to a Banach space Y is called an *ideal* in the Banach space $\mathcal{L}(X,Y)$ of all bounded linear operators if there exists a norm one projection P on $\mathcal{L}(X,Y)^*$ with $\ker P = \mathcal{K}(X,Y)^\perp$. Moreover, if there are $r,s \in (0,1]$ so that $\|f\| \geq r\|Pf\| + s\|f - Pf\|$ for all $f \in \mathcal{L}(X,Y)^*$, then we say that $\mathcal{K}(X,Y)$ is an $M(r,s)$-*ideal* in $\mathcal{L}(X,Y)$.

Well-studied M-ideals (see [HWW] for results and references) are precisely $M(1,1)$-ideals. From algebraic point of view, in case of C^*-algebras (but not in general), M-ideals are precisely closed two-sided algebraic ideals.

Our main goal is to show how $M(r,s)$-ideals of compact operators form new $M(r,s)$-ideals.

Theorem (compare [HJO]). *Let X and Y be Banach spaces. Assume that $\mathcal{K}(X,X)$ is an $M(r_1,s_1)$-ideal in $\mathcal{L}(X,X)$ with $r_1 + s_1/2 > 1$ and $\mathcal{K}(Y,Y)$ is an $M(r_2,s_2)$-ideal in $\mathcal{L}(Y,Y)$ with $r_2 + s_2/2 > 1$. Then $\mathcal{K}(X,Y)$ is an $M(r_1r_2, s_1s_2)$-ideal in $\mathcal{L}(X,Y)$.*

This talk is based on a joint work with Rainis Haller and Eve Oja.

References

[HJO] R. Haller, M. Johanson, E. Oja, *$M(r,s)$-inequality for $K(X,Y)$ in $L(X,Y)$*, Acta Comment. Univ. Tartuensis **11** (2007), 69−74.

[HWW] P. Harmand, D. Werner, W. Werner, *M-ideals in Banach Spaces and Banach Algebras*, Springer, Berlin, 1993.

On subalgebras of square of finite minimal majority algebra

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Irreducibility and reduction of nonlinear control systems: unification and extension via pseudo-linear algebra

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The talk is about applying the pseudo-linear algebra to unify the study of irreducibility and reduction for continuous- and discrete-time nonlinear control systems. The necessary and sufficient condition for irreducibility of nonlinear input-output equation is presented in terms of the common left factor of two polynomials describing the behaviour of the system. Besides unification, the tools of pseudo-linear algebra allow to extend the results for systems defined by difference, q-shift and q-difference operators.

Phylogenetic algebraic geometry

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Phylogenetic algebraic geometry studies algebraic varieties coming from statistical models of evolution. In some cases these models are toric and corresponding lattice polytopes can be studied. We explain how these varieties arise and give some new examples how lattice polytopes can solve problems in phylogenetic algebraic geometry.

Morita equivalence of semigroups

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A *Morita context* is a six-tuple $(S, T, {}_SP_T, {}_TQ_S, \theta, \phi)$, where S and T are semigroups, ${}_SP_T \in {}_S\mathsf{Act}_T$ and ${}_TQ_S \in {}_T\mathsf{Act}_S$ are biacts, and

$$\theta : {}_S(P \otimes_T Q)_S \to {}_SS_S, \quad \phi : {}_T(Q \otimes_S P)_T \to {}_TT_T$$

are biact morphisms such that, for every $p, p' \in P$ and $q, q' \in Q$,

$$\theta(p \otimes q)p' = p\phi(q \otimes p'), \quad q\theta(p \otimes q') = \phi(q \otimes p)q'.$$

Two monoids are called *Morita equivalent* if the categories of right acts over them are equivalent. It is known that two monoids are Morita equivalent if and only if there exists a Morita context with surjective mappings containing these monoids. The situation for semigroups is quite different from monoids, because these two conditions are not equivalent in general. Two semigroups are called *strongly Morita equivalent* if there exists a unitary Morita context with surjective mappings containing these monoids. This definition together with the assumption of "some kind of" local identity elements allows to develop quite an interesting Morita theory for semigroups. We give an overview of recent results in this area.

Some results on endomorphism monoids of medial quasigroups

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In this work we study the endomorphism semigroups of idempotent medial commutative quasigroups. Toyoda established a connection between medial quasigroups and Abelian groups (Theorem 2.10 in [1]). Motivated by Toyoda's result and some Puusemp's results on endomorphisms of Abelian groups, we study the endomorphisms of magmas which are "very close" to Abelian groups. To be more precise, we replace the associativity by a weaker assumption — mediality.

This talk is based on a joint work with Alar Leibak.

References

[1] Belousov, V. D. *Foundations of the theory of quasigroups and loops.* Moscow, Nauka, 1967 (in Russian).

Some results on rotationality of totally symmetric quasigroups

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A nonempty set Q with a binary operation $\cdot$ is called a *quasigroup* if for all $a, b \in Q$ the equations $ax = b$ and $ya = b$ have a unique solution. A quasigroup having an identity element e ($ex = x$, $xe = x$ for all $x \in Q$) is called a *loop*.

A quasigroup (loop) Q is called *totally symmetric* (briefly, TS-quasigroup (TS-loop)) if for all $x, y, z \in Q$ with $xy = z$ also all equations $yx = z$, $xy = z$, $zx = y$, $yz = x$, and $zy = x$ hold.

An element $x \in Q$ is called an *idempotent* if $xx = x$. A quasigroup is called *idempotent* if all its elements are idempotents. An idempotent TS-quasigroup is called *squas*. A TS-loop is called *sloop* if there are no idempotents except e.

We will consider finite quasigroups and let us denote by v the number of elements in Q and by a the number of idempotents of Q.

It is easy to prove that for $0 \leq a \leq v \in \mathbf{N}$ there exists a TS-quasigroup with v elements having a idempotents if and only if

- $v \equiv 0 \pmod 3$, $a \equiv 0 \pmod 3$ or
- $v \equiv 1, 5 \pmod 6$, $a \equiv 1 \pmod 3$, $a \neq v - 1$ or
- $v \equiv 2, 4 \pmod 6$, $a \equiv 1 \pmod 3$, $a \leq \frac{v}{2}$.

Therefore there exists a squas with v elements if and only if $v \equiv 1, 3 \pmod 6$ and there exists a sloop with v elements if and only if $v \equiv 2, 4 \pmod 6$.

Definition. Let $f \geq 0$ and $k > 0$ be integers. A TS-quasigroup (TS-loop) is said to be (f, k)-*rotational* if it admits an automorphism consisting of exactly f fixed points and k cycles of same length. The $(0, k)$-rotational TS-quasigroups (TS-loops) are often called *k-cyclic*.

In the talk we will discuss the possible values (the small ones only) for v and a for a TS-quasigroup (TS-loop) to be (f, k)-rotational.

Generalized syntactic semigroups of tree languages

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A new type of syntactic monoid and semigroup of tree languages is introduced. For each $n \geq 1$, we define for any tree language T its n-ary syntactic monoid $M^n(T)$ and its n-ary syntactic semigroup $S^n(T)$ as quotients of the monoid or semigroup, respectively, formed by certain new generalized contexts. For $n = 1$ these contexts are just the ordinary contexts, and $M^1(T)$ and $S^1(T)$ are the usual syntactic monoid and semigroup of T. For each $n \geq 1$, $M^n(T)$ and $S^n(T)$ are isomorphic to certain monoids and semigroups associated with the minimal tree recognizer of T. Using these syntactic monoids or semigroups, we can associate with any variety of finite monoids or semigroups, respectively, a variety of tree languages. Although there are varieties of tree languages that cannot be obtained this way, we prove that the definite tree languages can be characterized by the syntactic semigroups $S^2(T)$, which is not possible using the classical syntactic monoids or semigroups.

The lecture is based on joint work with *Antonio Cano Gómez*:

Antonio Cano Gómez and Magnus Steinby, Generalized contexts and n-ary syntactic semigroups of tree languages, TUCS Technical Report No. **968**, Turku Centre for Computer Science, Turku, March 2010

The computational complexity of solving equations over finite algebras

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It was always in the center of algebraic interest whether or not an equation can be solved. A similar problem is, whether or not two algebraic expressions agree at every substitution, or in other words whether or not an identity holds over an algebraic structure. In our talk we investigate the computational complexity of the above problems over finite classical algebraic strucures: groups, rings and semigroups. For finite structures the complexity of the two questions are in NP and coNP, respectively.

We show that for rings dichotomy holds, for example, checking an identity is decidable in polynomial time if the ring is nilpotent, and coNP-complete otherwise.

For groups the problem is more complicated, we show that for nilpotent and dihedral groups the problems are solvable in polynomial time and that for simple groups both problems are (co-)NP-complete. There are very few results known about solvable but non-nilpotent groups.

We show that extending an algerba with new operations might change the complexity. For example, for the alternating group $(A_4, \cdot, ^{-1})$ both problems are in P, but if we add the commutator as basic operation, then for the (extended) group $(A_4, \cdot, ^{-1}, [,])$ both problems become hard.