Nelson algebras of rough sets

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Let R be a quasiorder (reflexive and transitive binary relation) on a set U. We denote $R(x) = \{y \in U \mid x\, R\, y\}$ for $x \in U$. For any subset $X \subseteq U$, the *lower approximation* of X is $X^{\blacktriangledown} = \{x \in U \mid R(x) \subseteq X\}$ and the *upper approximation* of X is $X^{\blacktriangle} = \{x \in U \mid R(x) \cap X \neq \emptyset\}$. For any $X \subseteq U$, the pair $(X^{\blacktriangledown}, X^{\blacktriangle})$ is called the *rough set* of X. Let RS be the set of all rough sets. We denote by $\mathcal{RS}$ the set RS ordered coordinatewise, that is, $(X^{\blacktriangledown}, X^{\blacktriangle}) \leq (Y^{\blacktriangledown}, Y^{\blacktriangle})$ if and only if $X^{\blacktriangledown} \subseteq Y^{\blacktriangledown}$ and $X^{\blacktriangle} \subseteq Y^{\blacktriangle}$.

We showed in [1] that $\mathcal{RS}$ is an algebraic and completely distributive lattice, meaning that every element of $\mathcal{RS}$ can be expressed as a join of completely join-irreducible elements; completely join-irreducible elements of $\mathcal{RS}$ were also described.

In [2], we proved that on the lattice $\mathcal{RS}$ we can define a Nelson algebra $\mathbb{RS}$, and if $\mathbb{A}$ is a Nelson algebra defined on an algebraic lattice, then there exists a set U and a quasiorder R on U such that $\mathbb{A}$ is isomorphic to $\mathbb{RS}$.

As new observations it is shown that: (i) For *every* Nelson algebra $\mathbb{A}$, there exists a set U and a quasiorder R on U such that $\mathbb{A}$ is isomorphic to a subalgebra of $\mathbb{RS}$. (ii) For each quasiorder R, there exists a T_0-space $\mathcal{T}$ on the set $\mathcal{J}$ of all completely join-irreducible elements of $\mathbb{RS}$ such that the lattice $(\mathcal{T}, \cup, \cap)$ can be made to a Nelson algebra isomorphic to $\mathbb{RS}$.

Some illustrative examples of the above constructions are presented.

References

[1] Jouni Järvinen, Sándor Radeleczki, and Laura Veres, Rough sets determined by quasiorders, *Order* 26, 2009, 337–355.

[2] Jouni Järvinen and Sándor Radeleczki, Representation of Nelson algebras by rough sets determined by quasiorders, Algebra Universalis 66, 2011, 163–179.