

Number of solutions of some diagonal equations over finite fields

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We consider the diagonal equations $ax^n + by^n - cz^n = 1$ over finite fields. In general, the number of solutions of such equations may not be formulated. The solutions of such equations are closely related to the cyclotomic numbers considered first by Dickson in 1935.

Let F be a finite field of q elements, and $k \geq 2$ an integer such that $k \mid (q - 1)$. Let Φ be a multiplicative subgroup of F and $|\Phi| = k$. We will show our work in progress toward formulating the number of solutions of the mentioned equation under the following conditions.

- (a) $n = (q - 1)/k$;
- (b) $|\Phi \cap (\Phi a + b)| \leq 2$ for all $a, b \in F$, $a \notin \Phi \cup \{0\}$.

The condition (b) comes from the circular planar nearrings, which have connections to several geometrical and combinatorial objects, and have many interesting properties on its own right.

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References

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