Congruences of strongly Morita equivalent posemigroups

Valdis Laan

University of Tartu

Tartu, Estonia

A partially ordered semigroup (shortly posemigroup) S is said to have *common joint weak local units* if

$$(\forall s, s' \in S)(\exists u, v \in S)(s = usv \wedge s' = us'v).$$

Posemigroups S and T are called *strongly Morita equivalent* if there exist biposets $_S P_T, {_T}Q_S$ and surjective biposet morphisms $\theta : {_S}(P \otimes_T Q)_S \to {_S}S_S$, $\quad \phi : {_T}(Q \otimes_S P)_T \to {_T}T_T$ such that $SP = P, PT = P, TQ = Q, QS = Q$ and, for every $p, p' \in P$ and $q, q' \in Q$,

$$\theta(p \otimes q)p' = p\phi(q \otimes p'), \quad q\theta(p \otimes q') = \phi(q \otimes p)q'.$$

If s, s' are elements of a posemigroup S and ρ is an equivalence relation on S then we write $s \underset{\rho}{\leq} s'$ if there exist a natural number n and elements $s_1, \ldots, s_n \in S$ such that

$$s \leq s_1 \; \rho \; s_2 \leq s_3 \; \rho \; \ldots \; \rho \; s_n \leq s'.$$

A *posemigroup congruence* on a posemigroup S is a semigroup congruence ρ on it which satisfies the following condition: $s \underset{\rho}{\leq} s'$ and $s' \underset{\rho}{\leq} s$ implies $s \rho s'$.

Theorem. Congruence lattices of strongly Morita equivalent posemigroups with common joint weak local units are isomorphic.

Corollary. Congruence lattices of Morita equivalent pomonoids are isomorphic.

This talk is based on joint research with Tanel Tärgla.