

On the solidity of varieties of tree languages

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Hypersubstitutions (cf. [1, 3], for example) and tree homomorphisms (cf. [2], for example) are closely related notions; both map terms to terms by replacing function symbols with terms. In fact, we shall classify tree homomorphisms according to their underlying hypersubstitutions. Graczyńska and Schweigert (1990) defined a variety of algebras to be solid if it contains all derived algebras obtained from its members by hypersubstitutions. This notion was refined by Denecke and Reichel (1995) to M -solidity that restricts the requirement to a given monoid M of hypersubstitutions.

In this talk, which is based on [4], we consider general varieties of finite algebras (GVFAs) containing finite algebras of all finite types and general varieties of tree languages (GVTLs) that contain tree languages over all ranked alphabets. For a class \mathcal{K} of hypersubstitutions, \mathcal{K} -solid GVFA and \mathcal{K} -solid GVTLs are defined in such a way that for any so-called category of substitutions \mathcal{K} , a GVTL is \mathcal{K} -solid exactly in case the corresponding GVFA is \mathcal{K} -solid. We establish the solidity status of several known GVTLs with respect to certain categories of substitutions derived from some important classes of tree homomorphisms.

References

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