Workshop
“Algebra and its applications”

April 25-27, 2014

Mokko Farm, Änkküla village, Estonia

Abstracts

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Program

Friday, April 25:

16:30–16:40 Opening

16:40–18:40 Talks

- 16:40 Csaba Szabó, *Closed subgroups and reducts of homogeneous structures*
- 17:40 Nasir Sohail, *Closure, epimorphisms and dominions of ordered monoids*

18:45 Dinner

Saturday, April 26:

8:00 Breakfast

9:00–12:30 Talks

- 9:00 Jouni Järvinen, *Rough sets determined by tolerances*
- 10:00 Edmund Puczyłowski, *On the intersection graph of modules and rings*
- 11:00 Coffee break
- 11:30 Peter Mayr, *Computing in direct powers*
- 12:00 Oleg Košik, *Term-equivalence of semilattices*

12:30–13:30 Lunch

13:30–15:30 Discussions

15:30–18:30 Talks

- 15:30 David Karpuk, *Algebraic constructions for wireless channels and geodesic flow on Lie groups*
- 16:30 Coffee break
- 17:00 Ewa Pawłuszewicz, *Applications of functional universes in control theory*
- 17:30 Ülle Kotta, *Polynomial tools for discrete-time nonlinear control systems*
- 18:00 Tarmo Uustalu, *Coalgebraic update lenses*

19:00 Dinner

Sunday, April 27:

8:00 Breakfast

9:00–12:00 Talks

- 9:00 Kaie Kubjas, *Nonnegative matrix rank and the EM algorithm*
- 10:00 Rida-E Zenab, λ*-semidirect products via inductive categories*
- 10:30 Coffee break
- 11:00 Lauri Tart, *On congruence extension for Hamiltonian ordered algebras*
- 11:30 Valdis Laan, *Injective hulls for posemigroups*

12:00 Closing

12:10 Lunch

Rough sets determined by tolerances

Jouni Järvinen
Turku, Finland

In rough set theory, the idea is to approximate subsets of a given universe of discourse U in terms of the knowledge we have about the elements of U. Usually this knowledge is given in the form of a binary relation representing, for instance, indistinguishability, similarity, or preference among the objects in U. The *lower approximation* $X^{\blacktriangledown}$ of a subset X of U consists of the elements which *certainly* belong to X in terms of the knowledge we have, and $X^{\blacktriangle}$ is the set of elements *possibly* belonging to X.

This talk presents some new observations on rough sets defined by tolerance relations. These results are obtained in joint work with Sándor Radeleczki[1]. The term *tolerance relation* was introduced in the context of visual perception theory by E. C. Zeeman in the early 1960's motivated by the fact that indistinguishability of "points" in the visual world is limited by the discreteness of retinal receptors. One can argue that tolerances suit better for representing indistinguishability than equivalences, since transitivity often fails to hold: We may have a finite sequence of objects $x_1, x_2, \ldots, x_n$ such that each two consecutive objects x_i and x_{i+1} are indiscernible, but there is a notable difference between x_1 and x_n.

We show that for any tolerance R on U, the ordered sets of lower and upper rough approximations determined by R form isomorphic ortholattices. These ortholattices are completely distributive, thus forming atomistic Boolean lattices, if and only if R is induced by an irredundant covering of U, and in such a case, the atoms of these Boolean lattices are described.

The *rough set* of X is the pair $(X^{\blacktriangledown}, X^{\blacktriangle})$, and the set of all rough sets is $RS = \{(X^{\blacktriangledown}, X^{\blacktriangle}) \mid X \subseteq U\}$. We prove that the coordinatewise-ordered set RS of rough sets determined by a tolerance R on U is a complete lattice if and only if it is a complete subdirect product of the complete lattices of lower and upper rough approximations. We show that R is a tolerance induced by an irredundant covering of U if and only if RS is an algebraic completely distributive lattice, and in such a situation a Kleene algebra (and in fact a quasi-Nelson algebra) can be defined on RS.

[1] Jouni Järvinen & Sándor Radeleczki: Rough sets determined by tolerances, *International Journal of Approximate Reasoning* (in press).
`http://dx.doi.org/10.1016/j.ijar.2013.12.005`

Algebraic constructions for wireless channels and geodesic flow on Lie groups

David Karpuk
Aalto University
Helsinki, Finland

In this talk I will explain a few applications of algebra and number theory to communication over wireless channels. In particular, I will describe how to create and use number theoretic lattices suitable as codebooks for communication over wireless channels. Secondly, I will describe some recent work on how to slightly improve upon these methods by doing some basic numerical analysis on the Lie group $\mathrm{SO}(n)$.

Term-equivalence of semilattices

Oleg Košik
University of Tartu
Tartu, Estonia

A *term operation* of an algebra $\mathbf{A}$ is any operation on A that can be constructed from basic operations of $\mathbf{A}$ and projection maps. Two algebras are called *term-equivalent* if they have the same universes and the same term operations.

In our talk we will discuss term-equivalence of semilattices.

This talk is based on joint research with Peter Mayr.

Polynomial tools for discrete-time nonlinear control systems

Ülle Kotta
Institute of Cybernetics at Tallinn University of Technology
Tallinn, Estonia

Control systems research has a long history of mathematical rigor, with application to diverse branches of science and engineering. There is no question that nonlinear thinking can be of major benefit in practical problems, but the main concern of practitioners is that nonlinear methods, as a rule, require specialist knowledge to be understood and properly applied. It has been our purpose for some time by now to focus on development of theory, methods and algorithms that (i) mimic (in certain sense) their linear counterparts to be better accessible for potential users and (ii) are constructive and therefore suitable for implementation in symbolic software.

The algebraic approach, developed by us, uses differential forms (Kähler differentials) to describe the global (generic) linearization of nonlinear control systems. This description may be further rewritten by means of matrices over skew polynomial rings. When polynomial indeterminate is interpreted as a shift (or difference) operator, polynomial matrices act on differential one-forms, including the differentials of inputs and outputs, as operators and allow to represent the algorithms in a compact /transparent way, and sometimes even replace them by explicit formulas. Many important system properties (accessibility, irreducibility / reduction, realizability / realization) and control problem solutions (model matching, dynamic feedback linearization) have been made constructive in this setting.

The talk gives a short overview of polynomial methods, applicable for analytic nonlinear control systems. Though the focus of the talk is on discrete-time case, the approach works equally well in case of continuous-time systems. The additional benefit of the polynomial approach is that it allows to unify the study of continuous- and discrete-time systems, if combined with pseudo-linear algebra or time scale calculus.

Nonnegative matrix rank and the EM algorithm

Kaie Kubjas
Aalto University
Helsinki, Finland

We consider the mixture model of two discrete random variables, i.e. matrices of nonnegative rank at most r. The EM algorithm aims to maximize the log-likelihood function of the mixture model. In doing so, it approximates a matrix U with a matrix P of nonnegative rank at most r. We study a primary decomposition of the ideal of the EM fixed points and recognize the boundary components among its minimal primes.

This talk is based on joint work with Elina Robeva and Bernd Sturmfels.

Injective hulls for posemigroups

Valdis Laan
University of Tartu
Tartu, Estonia

We consider the category, where objects are partially ordered semigroups (posemigroups) and morphisms are order-preserving submultiplicative mappings, that is, mappings $f : S \to T$ with $f(s)f(s') \leq f(ss')$ for all $s, s' \in S$. Let $\mathcal{E}_{\leq}$ denote the class of all morphisms $h : S \to T$ in this category which are order-preserving, submultiplicative and satisfy the following condition: $h(s_1) \ldots h(s_n) \leq h(s)$ implies $s_1 \ldots s_n \leq s$ for all $s_1, \ldots, s_n, s \in S$.

It turns out that $\mathcal{E}_{\leq}$-injective objects in this category are quantales. We have also shown how to construct $\mathcal{E}_{\leq}$-injective hulls for a certain class of posemigroups. This class includes pomonoids, negatively ordered posemigroups with weak local units, linearly ordered cancellative posemigroups and upper semilattices with natural order.

This talk is based on joint research with Xia Zhang.

Computing in direct powers

Peter Mayr
Johannes Kepler University Linz
Linz, Austria

We will investigate the complexity of some computational problems in direct powers of algebraic structures. For an example, fix a finite algebra A (a group, a ring, a lattice,...). The Subpower Membership Problem for A is the following:

INPUT tuples $a_1, \ldots, a_k$ and b in A^n

PROBLEM Is b in the subalgebra of A^n generated by $a_1, \ldots, a_k$?

There are algebras known for which this decision problem is Exptime-complete. However Ross Willard observed that for groups and rings there is a polynomial-time algorithm based on classical computational group theory. In 2007 he asked whether the Subpower Membership Problem is in P for Mal'cev algebras in general. We give some partial results. The general question still remains open.

Applications of functional universes in control theory

Ewa Pawluszewicz
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Bialystok, Poland

A function universe is a set of partially defined functions that contains zero and is closed with respect to substitutions and amalgamations, see [1]. It can be seen as a generalization of a functional algebra.

From application point of view there are important such classes of function universes as global generating universes and differential/difference universes, [1, 2]. In control theory they can be used for solving the problem of equivalence of control systems, see [2, 4], or the realization problem, see [3].

References

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On the intersection graph of modules and rings

Edmund Puczyłowski
University of Warsaw
Warsaw, Poland

For a given module M, the intersection graph $G(M)$ of M is defined [1, 2] as the simple undirected graph whose set of vertices consists of non-trivial submodules of M and two distinct submodules N_1, N_2 are adjacent if and only if $N_1 \cap N_2 \neq 0$. For a given ring R the intersection graph of R is defined as $G({}_RR)$, where ${}_RR$ denotes the left R-module R. One of the first remarkable results in the area was obtained in [3], where an isomorphism problem on the intersection graph of finite abelian groups (regarded as modules over the ring of integers) was solved. From this result it in particular follows that if A, B are finite abelian p-groups for a prime p, then $A \simeq B$ if and only if $G(A) \simeq G(B)$. This result does not extend to all modules or even to all finite abelian groups but it shows that properties of the intersection graph of modules carry quite important information on modules and can be applied to study their structure. The aim of the talk is to present some results on the subject obtained in [1, 2, 4].

References

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Closure, epimorphisms and dominions of ordered monoids

Nasir Sohail
University of Tartu
Tartu, Estonia

Flatness and amalgamation properties of monoids are known to undergo severe restrictions after the introduction of order. In this talk I shall discuss some of my recent results showing that closure, dominions and epimorphisms of monoids are not affected in this way. I shall also revisit my recently proved zigzag theorem for partially ordered monoids.

Closed subgroups and reducts of homogeneous structures

Csaba Szabó
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The random graph, the rational numbers, the universal poset are widely investigated homogeneous structures. The first order definable reducts of these structures are in a one-to-one correspondence with the closed supergroups of their automorphism groups. Recently new techniques were developed to find these intermediate reducts. I will sketch a few ideas about how to find the reducts of a homogeneous structure and the corresponding subgroups.

On congruence extension for Hamiltonian ordered algebras

Lauri Tart
University of Tartu
Tartu, Estonia

An interesting question that has been studied for various algebraic structures (notably semigroups [1] and universal algebras [2]) is the following. Given an algebra $\mathcal{A}$, a subalgebra $\mathcal{B}$ of $\mathcal{A}$ and a congruence $\rho \in \mathsf{Con}(\mathcal{B})$, is it possible to find such a congruence $\theta \in \mathsf{Con}(\mathcal{A})$ that $\theta \cap (\mathcal{B} \times \mathcal{B}) = \rho$? If this can be done for every subalgebra $\mathcal{B}$ and every congruence $\rho \in \mathsf{Con}(\mathcal{B})$, then the algebra $\mathcal{A}$ is said to have the *congruence extension property* (CEP).

We have recently considered congruence extension in the context of ordered algebras. In this case there are several kinds of congruence extension properties (vanilla, lax, strong and strong lax) and the latter two have a strong influence on the structure of the ordered algebra $\mathcal{A}$. Specifically, we have to consider *strongly Hamiltonian algebras*, i.e. ordered algebras $\mathcal{A}$ which have only convex subalgebras, and all these subalgebras are congruence classes of some congruence on $\mathcal{A}$.

In this talk I will discuss the relationship between strong congruence extension and Hamiltonian algebras. The material is based on joint work with Valdis Laan and Nasir Sohail.

References

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Coalgebraic update lenses

Tarmo Uustalu
Institute of Cybernetics at TUT
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I will discuss bialgebraic/coalgebraic structures relevant for bidirectional programming.

Ordinary (asymmetric) *lenses* à la Foster et al. are a structure abstracting databases manipulable by means of operations of viewing (projecting the database into a view) and updating (merging a replacement view into the database). *Update lenses* are our refinement where updates are decoupled from views and the operation of merging a view is replaced with an operation of applying an update. The separately given set of updates must carry a monoid structure and act on the set of views.

Update lenses for a given set of views and monoid and action of updates are exactly bialgebras of a suitable functor and monad. But thanks to standard facts about distributive laws and liftings, they can also be described as matching pairs of coalgebras of two comonads, coalgebras of a single composite comonad etc.

Update lenses comodel the same (generally large) Lawvere theory that is modelled by algebras of what we have elsewhere called update monads, a refinement of state monads.

A dependently typed variation of update lenses has every view coming with its own set of updates.

This is joint work with Danel Ahman (University of Edinburgh, United Kingdom).

λ-semidirect products via inductive categories

Rida-E-Zenab
University of York
York, United Kingdom

The semidirect product of two inverse semigroups need not be inverse in general. Billhardt showed how to get round this difficulty by modifying the definition of semidirect product of two inverse semigroups to obtain what he termed a λ-*semidirect product* [1]. Billhardt later extended his construction, in the case where one component was a semilattice, to left ample semigroups [2]. Again, in this special case, this was extended further to the λ-semidirect product of a semilattice and a left restriction semigroup [3]. Wazzan found a new approach in the inverse case by first building an inductive groupoid [5].

We extend the above in two ways. First, we consider λ-semidirect product of arbitrary left restriction semigroups and find covering and embedding theorems of this λ-semidirect product. Using the notion of double actions taken from [4] we then introduce λ-semidirect product of (two-sided) restriction semigroups. Following Wazzan's technique we first construct an inductive category and then obtain the corresponding restriction semigroup.

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