Rough sets determined by tolerances

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In rough set theory, the idea is to approximate subsets of a given universe of discourse U in terms of the knowledge we have about the elements of U. Usually this knowledge is given in the form of a binary relation representing, for instance, indistinguishability, similarity, or preference among the objects in U. The *lower approximation* $X^{\blacktriangledown}$ of a subset X of U consists of the elements which *certainly* belong to X in terms of the knowledge we have, and $X^{\blacktriangle}$ is the set of elements *possibly* belonging to X.

This talk presents some new observations on rough sets defined by tolerance relations. These results are obtained in joint work with Sándor Radeleczki[1]. The term *tolerance relation* was introduced in the context of visual perception theory by E. C. Zeeman in the early 1960's motivated by the fact that indistinguishability of "points" in the visual world is limited by the discreteness of retinal receptors. One can argue that tolerances suit better for representing indistinguishability than equivalences, since transitivity often fails to hold: We may have a finite sequence of objects $x_1, x_2, \ldots, x_n$ such that each two consecutive objects x_i and x_{i+1} are indiscernible, but there is a notable difference between x_1 and x_n.

We show that for any tolerance R on U, the ordered sets of lower and upper rough approximations determined by R form isomorphic ortholattices. These ortholattices are completely distributive, thus forming atomistic Boolean lattices, if and only if R is induced by an irredundant covering of U, and in such a case, the atoms of these Boolean lattices are described.

The *rough set* of X is the pair $(X^{\blacktriangledown}, X^{\blacktriangle})$, and the set of all rough sets is $RS = \{(X^{\blacktriangledown}, X^{\blacktriangle}) \mid X \subseteq U\}$. We prove that the coordinatewise-ordered set RS of rough sets determined by a tolerance R on U is a complete lattice if and only if it is a complete subdirect product of the complete lattices of lower and upper rough approximations. We show that R is a tolerance induced by an irredundant covering of U if and only if RS is an algebraic completely distributive lattice, and in such a situation a Kleene algebra (and in fact a quasi-Nelson algebra) can be defined on RS.

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[1] Jouni Järvinen & Sándor Radeleczki: Rough sets determined by tolerances, *International Journal of Approximate Reasoning* (in press).
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