Injective hulls for posemigroups

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We consider the category, where objects are partially ordered semigroups (posemigroups) and morphisms are order-preserving submultiplicative mappings, that is mappings $f : S \to T$ with $f(s)f(s') \leq f(ss')$ for all $s, s' \in S$. Let $\mathcal{E}_{\leq}$ denote the class of all morphisms $h : S \to T$ in this category which are order-preserving, submultiplicative and satisfy the following condition: $h(s_1) \dots h(s_n) \leq h(s)$ implies $s_1 \dots s_n \leq s$ for all $s_1, \dots, s_n, s \in S$.

It turns out that $\mathcal{E}_{\leq}$-injective objects in this category are quantales. We have also shown how to construct $\mathcal{E}_{\leq}$-injective hulls for a certain class of posemigroups. This class includes pomonoids, negatively ordered posemigroups with weak local units, linearly ordered cancellative posemigroups and upper semilattices with natural order.

This talk is based on joint research with Xia Zhang.