On congruence extension for Hamiltonian ordered algebras

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An interesting question that has been studied for various algebraic structures (notably semigroups [1] and universal algebras [2]) is the following. Given an algebra $\mathcal{A}$, a subalgebra $\mathcal{B}$ of $\mathcal{A}$ and a congruence $\rho \in \mathsf{Con}(\mathcal{B})$, is it possible to find such a congruence $\theta \in \mathsf{Con}(\mathcal{A})$ that $\theta \cap (\mathcal{B} \times \mathcal{B}) = \rho$? If this can be done for every subalgebra $\mathcal{B}$ and every congruence $\rho \in \mathsf{Con}(\mathcal{B})$, then the algebra $\mathcal{A}$ is said to have the *congruence extension property* (CEP).

We have recently considered congruence extension in the context of ordered algebras. In this case there are several kinds of congruence extension properties (vanilla, lax, strong and strong lax) and the latter two have a strong influence on the structure of the ordered algebra $\mathcal{A}$. Specifically, we have to consider *strongly Hamiltonian algebras*, i.e. ordered algebras $\mathcal{A}$ which have only convex subalgebras, and all these subalgebras are congruence classes of some congruence on $\mathcal{A}$.

In this talk I will discuss the relationship between strong congruence extension and Hamiltonian algebras. The material is based on joint work with Valdis Laan and Nasir Sohail.

References

[1] X. Tang, Semigroups with the congruence extension property, *Semigroup Forum* **56** (1998), 228-264.

[2] E.W. Kiss, L. Márki, P. Pröhle, W. Tholen, Categorical algebraic properties. A compendium on amalgamation, congruence extension, epimorphisms, residual smallness, and injectivity, *Studia Sci. Math Hung.* **18** (1983), 79-141.