λ -semidirect products via inductive categories

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The semidirect product of two inverse semigroups need not be inverse in general. Billhardt showed how to get round this difficulty by modifying the definition of semidirect product of two inverse semigroups to obtain what he termed a λ -semidirect product [1]. Billhardt later extended his construction, in the case where one component was a semilattice, to left ample semigroups [2]. Again, in this special case, this was extended further to the λ -semidirect product of a semilattice and a left restriction semigroup [3]. Wazzan found a new approach in the inverse case by first building an inductive groupoid [5].

We extend the above in two ways. First, we consider λ -semidirect product of arbitrary left restriction semigroups and find covering and embedding theorems of this λ -semidirect product. Using the notion of double actions taken from [4] we then introduce λ -semidirect product of (two-sided) restriction semigroups. Following Wazzan's technique we first construct an inductive category and then obtain the corresponding restriction semigroup.

References

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