Workshop
"Algebra and its applications"

May 8-10, 2015

Marguse Holiday Centre
Nüpli village, Valga county
Estonia

Abstracts

List of Participants

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Program

Friday, 08.05:

13:00 Lunch
13:50–14:00 Opening
14:00–17:00 Talks
- 14:00 Joanna Grygiel, *Counting tolerances on chains*
- 14:30 Katarzyna Grygiel, *Enumeration of lambda terms: different models and approaches*
- 15:00 Coffee break
- 15:30 Kalle Kaarli, *Clones of compatible functions of abelian groups*
- 16:30 Lauri Tart, *On congruence extension for commutative pomonoids*

18:00 Dinner

Saturday, 09.05:

8:00 Breakfast
9:00–13:00 Talks
  9:00 Jānis Cīrulis, *Dependency of variables and orthomodular posets*
  10:00 Insa Krēmere, *Left-star order structure of Rickart *-rings*
  10:30 Coffee break
  11:00 Pēteris Daugulis, *New results in graph automorphism theory*
  12:00 Mart Abel, *Some results on spectral properties of algebras and the algebra of linear operators on an algebra*
13:00–14:00 Lunch
14:00–16:30 Talks
  14:00 Ülle Kotta, *Algebraic tools in nonlinear control*
  15:00 Coffee break
  15:30 Yu Kawano, *Algebraic geometric approach to accessibility and observability of polynomial control systems*
  16:00 Arvo Kaldmäe, *Feedback linearization of possibly non-smooth control systems using 'functions' algebra'*
20:00 Dinner

Sunday, 10.05:

8:00 Breakfast
9:00–12:00 Talks
  9:00 Oleg Košik, *Categorical equivalence of rings*
  9:30 Tarmo Uustalu, *Stateful runners for effectful computations*
  10:30 Coffee break
  11:00 Ülo Reimaa, *Wide right Morita contexts in bicategories with lax units*
  11:30 Valdis Laan, *Injective hulls for ordered algebras*
12:00 Closing
12:10 Lunch

Some results on spectral properties of algebras and the algebra of linear operators on an algebra

Mart Abel
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There are many books and papers written on the properties of the spectrum of an element in a Banach algebra or about the properties of the algebra of linear operators on a Banach algebra. Actually, these notions of spectrum and linear operators are purely algebraic and therefore it would be natural to study them in an algebraic way as far as possible.

In this talk we offer some generalisations of results known for Banach algebras. In our setting, the first task was to find out which properties of algebras were actually used in the proofs. The second task was to formulate these conditions in an algebraic matter and to try to obtain generalisations in an algebraic setting/language, including (arbitrary!) topology only when we were not able to avoid it. Perhaps the participants of the conference can recognise some results and give us some indications where similar results have been obtained in such or even in a more general setting.

Dependency of variables and orthomodular posets

Jānis Cīrulis
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This talk is expository: we show that the informal notion of dependency of variables leads to some well-tractable algebraic structures related to orthocomplemented and orthomodular posets. Also, various constructions and facts coming from the first field have their counterparts in the second one.

New results in graph automorphism theory

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Graph automorphism theory is an important branch of group representation theory dealing with group representations on binary relations (see [1], [2]). Although it has been advanced considerably in the last decades, there are unresolved questions related to structure and minimal graphs having given automorphism groups. We exhibit small graphs having alternating automorphisms groups and propositions related to semidirect automorphism groups.

References

[1] L. Babai (1995), Automorphism groups, isomorphism, reconstruction, In Graham, Ronald L.; Grotschel, Martin; Lovasz, Laszlo, Handbook of Combinatorics I, North-Holland, pp. 1447-1540.

[2] P. Cameron (2004) Automorphisms of graphs, in Topics in Algebraic Graph Theory (eds. L. W. Beineke and R. J. Wilson), Cambridge Univ. Press, Cambridge, (ISBN 0521801974), pp.137-155.

Counting tolerances on chains

Joanna Grygiel
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A tolerance relation of a lattice L is a reflexive and symmetric relation compatible with the operations of L. It is clear that every congruence of a lattice L is a tolerance on L. A tolerance T of a lattice L is called a glued tolerance if its transitive closure T^* is the total relation L^2.

We give the number of all tolerances and all glued tolerances on an n-element chain. We also estimate the density of congruences and glued tolerances among all tolerances in the case of chains.

This talk is based on joint work with Katarzyna Grygiel and Anetta Górnicka.

Enumeration of lambda terms: different models and approaches

Katarzyna Grygiel
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In recent years growing attention has been given to quantitative research in logic and computational models. Investigated objects (e.g., propositional formulae, tautologies, proofs, programs) can be seen as combinatorial structures, providing therefore the inspiration for combinatorists and computer scientists. In particular, several works have been devoted to studying properties of lambda calculus terms. From a practical point of view, the generation of random lambda terms is the core of debugging functional programs using random tests.

In my talk I will present several combinatorial models of lambda terms, as well as different attempts to solve the problem of enumerating closed terms. In most models, it is not difficult to define recurrence relations for the number of lambda terms of a given size. However, standard tools of analytic combinatorics are usually not sufficient to derive the asymptotic growth of the studied sequences since the related generating functions are expressed in the form of infinitely nested radicals.

References

[1] O. Bodini, D. Gardy, B. Gittenberger, A. Jacquot, Enumeration of generalized BCI lambda-terms, *Electronic Journal of Combinatorics* 20, 2013, pp. 4.

[2] R. David, K. Grygiel, J. Kozik, Ch. Raffalli, G. Theyssier, M. Zaionc, Asymptotically almost all lambda terms are strongly normalizing, *Logical Methods in Computer Science* 9(1:02), 2013, 1–30.

[3] K. Grygiel, P. Lescanne, Counting and generating lambda terms, *Journal of Functional Programming* 23(5), 2013, 594–628.

[4] K. Grygiel, P. Lescanne, Counting terms in the binary lambda calculus, *Discrete Mathematics & Theoretical Computer Science*, Proceedings of AofA, Paris, France, 2014, pp. 13.

Clones of compatible functions of abelian groups

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Clone is a set of finitary functions on a given set A which is closed with respect to composition and contains all projection maps. Given and algebra $\mathbf{A}$, the set of all term functions of $\mathbf{A}$ forms a clone which is denoted by $\mathrm{Clo}\mathbf{A}$. This is actually the clone generated by all basic operations of $\mathbf{A}$. The members of the clone generated by all basic operations and all constants of $\mathbf{A}$ are called *polynomial functions of* $\mathbf{A}$. This clone is denoted by $\mathrm{Pol}\mathbf{A}$. Obviously, all polynomial functions of $\mathbf{A}$ preserve all congruences of $\mathbf{A}$, or, in other words, are (congruence) *compatible*. Moreover, all compatible functions of $\mathbf{A}$ form a clone denoted by $\mathrm{Comp}\mathbf{A}$. Thus, we have the following inclusions, for any algebra $\mathbf{A}$:

$$\mathrm{Clo}\mathbf{A} \subseteq \mathrm{Pol}\mathbf{A} \subseteq \mathrm{Comp}\mathbf{A}\,.$$

Recall that an algebra $\mathbf{A}$ is called *affine complete*, if $\mathrm{Pol}\mathbf{A} = \mathrm{Comp}\mathbf{A}$.

For any class $\mathcal{V}$ of algebras we introduce three subclasses:

- $\mathcal{V}_1$ – the class of all $\mathbf{A} \in \mathcal{V}$ such that $\mathrm{Comp}\mathbf{A}$ is finitely generated;
- $\mathcal{V}_2$ – the class of all $\mathbf{A} \in \mathcal{V}$ such that $\mathrm{Comp}\mathbf{A}$ is generated by a finite subset plus all constants;
- $\mathcal{V}_3$ – the class of all $\mathbf{A} \in \mathcal{V}$ such that $\mathrm{Comp}\mathbf{A}$ is generated by $\mathrm{Comp}_n\mathbf{A}$, for some positive integer n.

Obviously, the following inclusions hold: $\mathcal{V}_1 \subseteq \mathcal{V}_2 \subseteq \mathcal{V}_3$.

The aim of the present work was to describe the classes $\mathcal{V}_i$, $i = 1, 2, 3$, if $\mathcal{V}$ is the variety of abelian groups. The work is not finished yet. As an example, we present the following result.

Theorem. Let $\mathcal{V}$ be the class of all abelian groups. Then $\mathbf{A} \in \mathcal{V}_1$ if and only if $\mathbf{A}$ is finitely generated and affine complete or $\mathbf{A}$ is the direct sum of two finite groups of coprime exponents, one of them cyclic and the other affine complete.

This talk is based on joint work with Erhard Aichinger (Linz).

Feedback linearization of possibly non-smooth control systems using 'functions' algebra'

Arvo Kaldmäe
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An algebraic method called 'functions' algebra', which is based on the algebra of partitions [1], will be used to linearize a discrete-time control system by state feedback and change of coordinates. The advantage of this method over other existing methods is that it allows also to consider systems described by non-smooth functions. The main objects we work with are vector functions. These vector functions are divided into equivalence classes on the basis of a partial preorder, which defines an equivalence relation. The partial preorder acts on the set of equivalence classes as a partial order and thus the set of equivalence classes becomes a lattice. This lattice is connected to the system description

$$\begin{aligned} x(t+1) &= f(x(t), u(t)) \\ x(t) &\in \mathbb{R}^n, \quad u(t) \in \mathbb{R}^m, \end{aligned}$$

by certain binary relation Δ. Necessary and sufficient conditions are found for the existence of a coordinate transformation $z(t) = \varphi(x(t))$ and a static state feedback $u(t) = G(x(t), v(t))$, $v(t) \in \mathbb{R}^m$, such that in coordinates $z(t)$, after applying the feedback, the system equations are linear.

References

[1] J. Hartmanis and R. Stearns, *The algebraic structure theory of sequential machines*, Prentice-Hall, New York, 1966.

Algebraic geometric approach to accessibility and observability of polynomial control systems

Yu Kawano
Kyoto University
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In control theory, accessibility and observability are fundamental concepts, which can be interpreted as surjectivity and injectivity of some iterated mappings, respectively. For nonlinear control systems, one of the common problems for verifying these properties is that the sufficient number of iterations is unclear. In this talk, on the basis of results of algebraic geometry, I will show that for polynomial control systems, accessibility and observability can be verified in finite time.

Categorical equivalence of rings

Oleg Košik
University of Tartu
Tartu, Estonia

A variety of algebras can be considered as a category in a natural way: the objects are the algebras in the variety, and the morphisms are the homomorphisms between them. Two algebras **A** and **B** are called *categorically equivalent*, if the varieties they generate are equivalent as categories, and the equivalence functor maps **A** to **B**.

In this talk I will present some recent results about categorical equivalence of rings with unity.

The talk is based on a joint work with Kalle Kaarli and Tamás Waldhauser (Szeged, Hungary).

Algebraic tools in nonlinear control

Ülle Kotta
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The most popular approaches in nonlinear control, besides stabilization and optimization methods, are those based on differential geometry and differential (or difference) algebra. The second one, developed further, uses differential 1-forms (or, alternatively, Kähler differentials) to describe the generic linearization of nonlinear control system. Many important system properties and control problem solutions have been made constructive within this setting.

In the talk we demonstrate the application of this approach on three problems in nonlinear control. First, we study the accessibility property, which is a system analysis problem. Second, we study an iconic problem of feedback linearization, which is a synthesis problem. Third, we study realization of input-output equation in the state space form, which is a modeling problem. These problems were chosen since the necessary and sufficient (solvability) conditions are based on the same sequence of vector spaces of differential 1-forms over the difference field of meromorphic functions. Note that the shift operator in this field is defined using the difference equations that define the nonlinear control system.

The algorithms related to these problems (and many others) have been implemented in the package NLControl, developed in the computer algebra system Mathematica (www.nlcontrol.ioc.ee). The functions in the package are unique in the sense that at present there does not exist any worldwide alternative software to solve the same problems. A web interface has been developed for this package so that anyone can use it with only an internet browser.

Left-star order structure of Rickart *-rings

Insa Krēmere
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Janowitz proved in 1983 that the initial segments of a Rickart *-ring with the star order are orthomodular posets. We prove the same for the left-star order $* \leq$, which was introduced by Marovt et al., by finding an orthogonality relation which corresponds to $* \leq$ in a certain way (makes the ring a quasi-orthomodular poset) and then applying a result proved by Cīrulis which states that the initial segments of any quasi-orthomodular poset are orthomodular.

Injective hulls for ordered algebras

Valdis Laan
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Let $\mathcal{C}$ be a category and let $\mathcal{M}$ be a class of morphisms in $\mathcal{C}$. An object Q from $\mathcal{C}$ is called $\mathcal{M}$-*injective* in $\mathcal{C}$ if for any morphism $h : A \to B$ in $\mathcal{M}$ and any morphism $f : A \to Q$ in $\mathcal{C}$ there exists a morphism $g : B \to Q$ in $\mathcal{C}$ such that $gh = f$.

We consider injectivity in categories, where objects are ordered algebras of the same type and morphisms are monotone mappings $f : A \to B$ such that

$$\omega_B(f(a_1), \dots, f(a_n)) \leqslant f(\omega_A(a_1, \dots, a_n))$$

for every n-ary operation ω and

$$\omega_B \leqslant f(\omega_A)$$

for every nullary operation ω. As $\mathcal{M}$ we use a class of specific order-embeddings. It turns out that $\mathcal{M}$-injective objects in such categories are precisely sup-algebras. An ordered algebra is called a *sup-algebra* if its underlying poset is a complete lattice and its operations are compatible with joins (suprema). Many quantale-like structures are examples of sup-algebras.

For ordered algebras satisfying certain assumption one can also give a construction of injective hulls.

This talk is based on joint research with Xia Zhang.

Wide right Morita contexts in bicategories with lax units

Ülo Reimaa
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The concept of Morita contexts between unital rings or monoids can be generalized using the concept of *wide right Morita contexts* [1] in bicategories. For Morita contexts between semigroups, we need to assume some things about the semigroups and the Morita contexts in order to get a theory similar to the case of monoids [2].

We study how some of the assumptions on semigroups and Morita contexts in [2] can be translated into the bicategorical setting. In order to do that, we define wide right Morita contexts in *lax-unital bicategories* and give a few generalized results that this definition allows for.

References

[1] L. El Kaoutit. Wide morita contexts in bicategories, *Arab. J. Sci. Eng.* 33, 2008, 153–173.

[2] M. V. Lawson, Morita equivalence of semigroups with local units, *J. Pure Appl. Algebra* 215, 2011, 455–470.

On congruence extension for commutative pomonoids

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We continue our investigation of congruence extension in the case of ordered algebras. Recall that an ordered algebra $\mathcal{A}$ has the *congruence extension property* (CEP) with respect to a subalgebra $\mathcal{B}$ if every order-congruence θ on $\mathcal{B}$ is the restriction of some order-congruence Θ on $\mathcal{A}$, i.e. $\Theta \cap (B \times B) = \theta$. To be more specific, we have established some results about (strong, quasiorder) congruence extension for groups and commutative pomonoids.

The talk is based on joint work with Valdis Laan and Nasir Sohail.

References

[1] V. Laan, N. Sohail, L. Tart, Hamiltonian ordered algebras and congruence extension, submitted.

Stateful runners for effectful computations

Tarmo Uustalu
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This talk is about Moggi's [1] monad-based approach to effectful computation, as improved by Plotkin and Power [2], who started to look at the Lawvere theories giving rise to the relevant monads.

We consider the question of what structure is required of a set so that computations in a given notion of computation can be run statefully with this set as the state set. For running nondeterministic computations statefully, a resolver structure is needed; for interactive input/output computations, a "responder-listener" structure is necessary; to be able to serve stateful computations, the set must carry the structure of a lens.

We show that, in general, to be a stateful runner of computations for a monad corresponding to a Lawvere theory (defined as a set equipped with a monad morphism between the given monad and the state monad for this set) is the same as to be a comodel of the theory, i.e., a coalgebra of the corresponding comonad.

We work out a number of instances of this observation and also compare runners to Plotkin and Pretnar's [3] effect handlers.

References

[1] E. Moggi. Notions of computation and monads. *Inf. and Comput.*, v. 93, n. 1, pp. 55–92, 1991.

[2] G. Plotkin, J. Power. Notions of computation determine monads. In M. Nielsen, U. Engberg, eds., *Proc. of 5th Int. Conf. on Foundations of Software Science and Computation Structures, FoSSaCS 2002 (Grenoble, Apr. 2002)*, *Lect. Notes in Comput. Sci.*, v. 2303, pp. 342–356. Springer, 2002.

[3] G. Plotkin, M. Pretnar. Handling algebraic effects. *Log. Methods in Comput. Sci.*, v. 9, n. 4, article 23, 2013.