Clones of compatible functions of abelian groups

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Clone is a set of finitary functions on a given set A which is closed with respect to composition and contains all projection maps. Given and algebra $\mathbf{A}$, the set of all term functions of $\mathbf{A}$ forms a clone which is denoted by $\mathrm{Clo}\mathbf{A}$. This is actually the clone generated by all basic operations of $\mathbf{A}$. The members of the clone generated by all basic operations and all constants of $\mathbf{A}$ are called *polynomial functions of* $\mathbf{A}$. This clone is denoted by $\mathrm{Pol}\mathbf{A}$. Obviously, all polynomial functions of $\mathbf{A}$ preserve all congruences of $\mathbf{A}$, or, in other words, are (congruence) *compatible*. Moreover, all compatible functions of $\mathbf{A}$ form a clone denoted by $\mathrm{Comp}\mathbf{A}$. Thus, we have the following inclusions, for any algebra $\mathbf{A}$:

$$\mathrm{Clo}\mathbf{A} \subseteq \mathrm{Pol}\mathbf{A} \subseteq \mathrm{Comp}\mathbf{A}\,.$$

Recall that an algebra $\mathbf{A}$ is called *affine complete*, if $\mathrm{Pol}\mathbf{A} = \mathrm{Comp}\mathbf{A}$.

For any class $\mathcal{V}$ of algebras we introduce three subclasses:

$\mathcal{V}_1$ – the class of all $\mathbf{A} \in \mathcal{V}$ such that $\mathrm{Comp}\mathbf{A}$ is finitely generated;

$\mathcal{V}_2$ – the class of all $\mathbf{A} \in \mathcal{V}$ such that $\mathrm{Comp}\mathbf{A}$ is generated by a finite subset plus all constants;

$\mathcal{V}_3$ – the class of all $\mathbf{A} \in \mathcal{V}$ such that $\mathrm{Comp}\mathbf{A}$ is generated by $\mathrm{Comp}_n\mathbf{A}$, for some positive integer n.

Obviously, the following inclusions hold: $\mathcal{V}_1 \subseteq \mathcal{V}_2 \subseteq \mathcal{V}_3$.

The aim of the present work was to describe the classes $\mathcal{V}_i$, $i = 1, 2, 3$, if $\mathcal{V}$ is the variety of abelian groups. The work is not finished yet. As an example, we present the following result.

Theorem. Let $\mathcal{V}$ be the class of all abelian groups. Then $\mathbf{A} \in \mathcal{V}_1$ if and only if $\mathbf{A}$ is finitely generated and affine complete or $\mathbf{A}$ is the direct sum of two finite groups of coprime exponents, one of them cyclic and the other affine complete.

This talk is based on joint work with Erhard Aichinger (Linz).