Injective hulls for ordered algebras

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Let $\mathcal{C}$ be a category and let $\mathcal{M}$ be a class of morphisms in $\mathcal{C}$. An object Q from $\mathcal{C}$ is called $\mathcal{M}$-*injective* in $\mathcal{C}$ if for any morphism $h : A \to B$ in $\mathcal{M}$ and any morphism $f : A \to Q$ in $\mathcal{C}$ there exists a morphism $g : B \to Q$ in $\mathcal{C}$ such that $gh = f$.

We consider injecitvity in categories, where objects are ordered algebras of the same type and morphisms are monotone mappings $f : A \to B$ such that

$$\omega_B(f(a_1), \ldots, f(a_n)) \leqslant f(\omega_A(a_1, \ldots, a_n))$$

for every n-ary operation ω and

$$\omega_B \leqslant f(\omega_A)$$

for every nullary operation ω. As $\mathcal{M}$ we use a class of specific order-embeddings. It turns out that $\mathcal{M}$-injective objects in such categories are precisely sup-algebras. An ordered algebra is called a *sup-algebra* if its underlying poset is a complete lattice and operations are compatible with joins (suprema). Many quantale-like structures are examples of sup-algebras.

For ordered algebras satisfying certain assumption one can also give a construction of injective hulls.

This talk is based on joint research with Xia Zhang.