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“Algebra and its applications”

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Abstracts

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Program

Friday, 05.05:

13:00 Lunch
13:50–14:00 Opening
14:00–18:00 Talks
- 14:00 Sándor Radeleczki, *Congruence lattices of algebras on a finite set*
- 15:00 Jouni Järvinen, *Representing regular Kleene algebras in terms of rough sets*
- 16:00 Coffee break
- 16:30 Mati Abel, *When the division topological algebra is trivial*
- 17:00 Mart Abel, *Topological Segal algebras*
- 17:30 Reyna María Pérez-Tiscareño, *About locally m-convex algebras with dense finitely generated ideals*

18:00 Dinner
19:00 Round Table

Saturday, 06.05:

8:00 Breakfast
9:00–12:15 Talks
- 9:00 Kalle Kaarli, *On categorical equivalence of finite commutative rings*
- 9:30 Peeter Puusemp, *Some results on endomorphism semigroups of groups*
- 10:30 Coffee break
- 10:45 Jānis Cīrulis, *On focal Baer semigroups*
- 11:45 Insa Cremer, *Rickart rings and skew nearlattices*

12:30 Lunch
13:30–15:45 Talks
- 13:30 Tarmo Uustalu *Interaction morphisms and Turing computation*
- 14:00 Niccolò Veltri, *Coherence for skew near-semiring categories*
- 14:30 Coffee break
- 14:45 Silvio Capobianco, *Cellular automata on Cayley graphs: an overview*

16:00-19:00 Free time
19:00 Dinner
19:30 Short Excursion

Sunday, 7.05:

8:00 Breakfast
9:00–12:15 Talks
    9:00 Pēteris Daugulis, *Classifying nonequivalent presentations of finite groups - some special cases and series*
    9:55 Valdis Laan, *Fair semigroups*
    10:20 Lauri Tart, *On Morita equivalence of fair semigroups*
    10:45 Coffee break
    10:55 Laur Tooming, *Morita invariants of semirings*
    11:25 Ülo Reimaa, *Morita theorems for firm categorical semigroups*
11:55 Closing
12:00 Lunch

Topological Segal algebras

Mart Abel
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Segal algebras have been in use for more than 50 years, starting in the context of subalgebras of $L^1(G)$ with G a locally compact group and settling in the context of Banach algebras about 20 years ago. During the last 5 years, the scope of Segal algebras has broadened to the classes of Fréchet or locally multiplicatively convex (topological) algebras. In this talk we offer an overview of the different classes of Segal algebras used before and propose a generalisation of the definition of Segal algebra to the class of arbitrary (real or complex) topological algebras, introduced in [1] and [2], with some recent results.

References

[1] M. Abel, Generalisation of Segal algebras for arbitrary topological algebras (manuscript, submitted).

[2] M. Abel, About the density property in the space of continuous maps vanishing at infinity (manuscript, submitted).

When the division topological algebra is trivial

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Conditions, that a topological division algebra A over $\mathbb{C}$ is trivial (that is, $A = \mathbb{C}e$, where e is the unit element of A), are given. A new model of the micro world by this result is introduced.

Cellular automata on Cayley graphs: An overview

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A cellular automaton (CA) on a group G is an endomorphism of the dynamical system (S^G, σ) where S is a finite set of states and σ is the action of G on the space S^G of configurations (global states) by left multiplication. A famous theorem by Curtis, Lyndon and Hedlund characterizes CA as those transformations of S^G obtainable by synchronous update of identical finite-state automata, whose input alphabet is the set of patterns (local states) on a neighborhood defined by finitely many fixed offsets.

In this expository talk we illustrate the basics of classical cellular automata theory and discuss how some of the most famous results depend on specific properties of the underlying groups. A special focus will be given to the Garden of Eden theorem, which states that, for CA on a specific class of groups, surjectivity is mutually exclusive with the ability to correct finitely many errors in finite time.

References

[1] L. Bartholdi, Gardens of Eden and amenability on cellular automata, *J. Eur. Math. Soc.* 12, 2010, 241-248.

[2] S. Capobianco, P. Guillon, J. Kari, Surjective cellular automata far from the Garden of Eden, *Disc. Math. Theor. Comp. Sci.* 15, 2013, 41-60.

[3] T.G. Ceccherini-Silberstein, M. Coornaert, *Cellular Automata and Groups*, Springer, 2010.

[4] G.A. Hedlund, Endomorphisms and automorphisms of the shift dynamical system. *Math. Syst. Th.* 3, 1969, 320-375.

[5] J. Kari, Theory of cellular automata: A survey, *Theor. Comp. Sci.* 334, 2005, 3-33.

On focal Baer semigroups

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We aim to give some general information about Baer semigroups, to specify a subclass of them, and to show that several significant characteristics of star-ordered Rickart *-rings can be transferred to semigroups in this subclass. A focal Baer semigroup S is a semigroup with 0 expanded by two unary idempotent-valued operations, ${}^{\backprime}$ and ${}'$, such that the left (right) ideal generated by $x^{\backprime}$ (resp., x') is the left (resp., right) annihilator of x. S is said to be equifocal if the ranges of both operations coincide and $p^{\backprime} = p'$ for every p from the common range P. Such a semigroup is shown to be P-semiabundant. If S is also Lawson reduced, then P is an orthomodular lattice (under the standard order of idempotents), and a modified version of Drazin's star partial order turns out to be the natural order on S. Under this order, S is a partial lower semilattice, and every initial segment of S is order isomorphic to a subortholattice of P.

Rickart rings and skew nearlattices

Insa Cremer
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Rīga, Latvia

Sussman and Subrahmayam proved in [1] and [2] that a certain kind of reduced ring (called *m-domain ring* in [2]) can be decomposed into a collection of disjoint subsets which are closed with respect to multiplication. In [3] it is shown that reduced Rickart rings and m-domain rings are the same thing. This talk is about the order structure of a reduced Rickart ring's decomposition into disjoint semigroups.

Cīrulis proved in [4] that every right normal skew nearlattice can be regarded as a structure called a *strong semilattice of semigroups*, and in [5] he shows that any reduced Rickart ring admits the structure of a right normal skew nearlattice. It turns out that this strong semilattice of semigroups arises from the semigroup decomposition of [2].

References

[1] I. Sussman, Ideal structure and semigroup domain decomposition of associate rings, *Mathematische Annalen* 140(2), 1960, 87-93.

[2] N. V. Subrahmanyam, Structure theory for a generealized Boolean ring, *Mathematische Annalen* 141(4), 1960, 297-310.

[3] J. Cīrulis and I. Cremer, Notes on reduced Rickart rings, to be published soon.

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[5] J. Cīrulis, Relatively orthocomplemented skew nearlattices in Rickart rings, *Demonstratio Mathematica* 48(4), 2015, 493-508.

Classifying nonequivalent presentations of finite groups - some special cases and series

Peteris Daugulis
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Daugavpils, Latvia

The problem of classifying equivalence classes of presentations up to isomorphism of Cayley graphs is considered in this talk. We give definitions and discuss computational results. We find all equivalence classes of presentations of dicyclic groups having two generators. An innovative technique for solving such problems is used. These results may be used in characterizing group structure and properties.

References

[1] Bosma, W., Cannon, J., and Playoust, C. (1997), The Magma algebra system. I. The user language, *J. Symbolic Comput.*, 24, pp. 235-265.

[2] Robinson, D (1996) A course in the theory of groups. Springer-Verlag, New York.

[3] Rotman, J (1995) An introduction to the theory of groups. Springer-Verlag, New York.

Representing regular Kleene algebras in terms of rough sets

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This talk describes a joint work with Sándor Radeleczki [1].

A collection $\mathcal{H}$ of nonempty subsets of U is called a *covering* of U if $\bigcup \mathcal{H} = U$. A covering $\mathcal{H}$ is *irredundant* if $\mathcal{H} \setminus \{X\}$ is not a covering for any $X \in \mathcal{H}$. A *tolerance* is a reflexive and symmetric binary relation. Each covering $\mathcal{H}$ *induces* a tolerance $R_{\mathcal{H}} = \bigcup\{X^2 \mid X \in \mathcal{H}\}$.

For any binary relation R on U, the *lower approximation* of a subset X of U is $X^{\blacktriangledown} = \{x \in U \mid R(x) \subseteq X\}$ and X's *upper approximation* is $X^{\blacktriangle} = \{x \in U \mid R(x) \cap X \neq \emptyset\}$, where $R(x) = \{y \in U \mid x\,R\,y\}$. The set of *rough sets* is $\mathcal{RS} = \{(X^{\blacktriangledown}, X^{\blacktriangle}) \mid X \subseteq U\}$.

A *De Morgan algebra* $(L, \vee, \wedge, \sim, 0, 1)$ is a bounded distributive lattice with an operation $\sim$ satisfying $\sim\sim x = x$ and $x \leq y$ iff $\sim y \leq \sim x$. A *Kleene algebra* is a De Morgan algebra in which $x \wedge \sim x \leq y \vee \sim y$ holds.

A double pseudocomplemented lattice $(L, \vee, \wedge, ^*, ^+, 0, 1)$ is called *regular* if $x^* = y^*$ and $x^+ = y^+$ imply $x = y$. If a De Morgan algebra is such that its underlying lattice is pseudocomplemented, then it forms a double pseudocomplemented lattice where $x^+ = \sim(\sim x)^*$. We say that a De Morgan algebra (or a Kleene algebra) is *regular* if its underlying lattice is a regular double pseudocomplemented lattice. Note that a De Morgan algebra defined on an algebraic lattice is always a double pseudocomplemented lattice.

It is known that if $\mathcal{RS}$ is determined by a tolerance induced by an irredundant covering, then $\mathcal{RS}$ forms an algebraic lattice and determines a regular Kleene algebra. We show how any regular Kleene algebra defined on an algebraic lattice is isomorphic to a rough set Kleene algebra defined by a tolerance induced by an irredundant covering.

References

[1] J.Järvinen, S. Radeleczki: Representing regular Kleene algebras by tolerance-based rough sets, arXiv:1610.09847 (submitted to a journal).

On categorical equivalence of finite commutative rings

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This is a joint research with Tamás Waldhauser (Szeged, Hungary).

The (universal) algebras $\mathbf{A}$ and $\mathbf{B}$ are said to be *categorically equivalent* if there is an equivalence functor F between varieties generated by them such that $F(\mathbf{A}) = \mathbf{B}$. The algebras $\mathbf{A}$ and $\mathbf{B}$ are said to be *term equivalent* if they have the same universe and their clones of term functions coincide. It is easy to see that term equivalent algebras are categorically equivalent.

A well known result by C. Bergman and J. Berman says that for any primes p and q and any natural number n, the Galois fields $\mathrm{GF}(p^n)$ and $\mathrm{GF}(q^n)$ are categorically equivalent [1]. We proved in [2] that in the case of finite rings semisimplicity is a categorical property. Moreover, we showed that if two finite semisimple rings are categorically equivalent then this follows from the aforenamed result of C. Bergman and J. Berman.

On the other hand, in [2] we could not find non-trivial examples of categorically equivalent finite non-semisimple rings. We proved that if two finite non-semisimple rings of prime power characteristic are categorically equivalent then their characteristics coincide.

In the present work we have focused on the commutative case. Our basic results are the following.

1. Two categorically equivalent finite commutative rings of the same prime power characteristic have isomorphic additive groups. Thus, in particular, they have the same order.
2. For every odd prime p there exist non-isomorphic but term equivalent rings of order p^3.
3. Let $\mathbf{R}$ and $\mathbf{S}$ be categorically equivalent finite commutative rings of prime characteristic p such that $\mathbf{R}$ is generated by one element over a subfield of $\mathbf{R}$. Then $\mathbf{R}$ and $\mathbf{S}$ are isomorphic.

Problem Is it true that categorically equivalent finite commutative non-semisimple rings with the same universe are necessarily term equivalent?

References

[1] C. Bergman, J. Berman, Morita equivalence of almost primal clones I, *J. Pure Appl. Algebra* 108, 1996, 175–201.

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Fair semigroups

Valdis Laan
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Fair semigroups are non-additive analogues of xst-rings, introduced by Xu, Shum and Turner-Smith [2].

If S is a semigroup then a right S-act A_S is called *unitary* if $AS = A$. We say that a semigroup S is a *right fair semigroup* (see [1]) if every subact of a unitary right S-act is unitary. One defines left fair semigroups dually. By a *fair semigroup* we mean a semigroup which is both left and right fair.

It turns out that a semigroup S is right fair if and only if for every sequence $(s_i)_{i\in\mathbb{N}} \in S^{\mathbb{N}}$ of elements of S there exist $n \in \mathbb{N}$ and $u \in S$ such that

$$s_n \ldots s_2 s_1 u = s_n \ldots s_2 s_1.$$

We will give a list of examples of fair semigroups and some basic facts about them.

This talk is based on joint research with László Márki.

References

[1] V. Laan, L. Márki, Fair semigroups and Morita equivalence, *Semigroup Forum* 92, 2016, 633-644.

[2] Y. H. Xu, K. P. Shum, R. F. Turner-Smith, Morita-like equivalence of infinite matrix subrings, *J. Algebra* 159, 1993, 425–435.

About locally m-convex algebras with dense finitely generated ideals

Reyna María Pérez-Tiscareño
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It is known, as a consequence of a theorem of Richard Arens, that a commutative Fréchet locally m-convex algebra E with unit, does not have dense finitely generated ideals. In this talk we shall see that this result can fail to hold if E is not complete and metrizable.

Some results on endomorphism semigroups of groups

Peeter Puusemp
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I consider the following problems on endomorphism semigroups of groups:

- endomorphism semigroups of Abelian groups;
- some embedding theorems in connection with endomorphism semigroups of groups;
- endomorphism semigroups of wreath products of groups.

Congruence lattices of algebras on a finite set

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This is joint work with R. Pöschel and D. Jakubíková-Studenovská.

The congruence lattices of all algebras defined on a fixed finite set A ordered by inclusion form a finite atomistic lattice $\mathcal{E}$. We describe the atoms and coatoms of this lattice, based on some results of [1] and [2]. Using the description of some particular meet-irreducible elements we deduce several properties of the lattice $\mathcal{E}$; in particular, we prove that $\mathcal{E}$ is tolerance-simple whenever $|A| \geq 4$.

References

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Morita theorems for firm categorical semigroups

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The Morita theory of rings can be thought of as the study of equivalences between categories of modules, or equivalently, as the study of invertible bimodules.

The the additive structure of rings is not necessary to get a nice theory of Morita equivalence. When we forget the additive structure, we get the Morita theory of monoids. Indeed, a good theory of Morita equivalence is known to exist for categorical monoids, meaning monoids on objects M of some category $\mathcal{C}$, with multiplication maps $M \otimes M \to M$, where $- \otimes -$ is a (tensor) product functor on $\mathcal{C}$. The special case of rings arises when $- \otimes -$ is the tensor product functor on the category of abelian groups.

We outline how to get a similar theory of Morita equivalence for *firm categorical semigroups*, which are categorical semigroups that can act as a unit with respect to the tensor product of bimodules. We will illustrate the results with examples from topology.

On Morita equivalence of fair semigroups

Lauri Tart
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Let S be a semigroup. A right S-act A_S is called *unitary* if $AS = A$. A semigroup S is called a *right fair semigroup* (see [1]) if every subact of a unitary right S-act is unitary. Left fair semigroups are defined dually and a *fair semigroup* is one that is both left and right fair. Every fair semigroup S contains a subsemigroup S' with weak local units, called its *unitary part*.

Semigroups S and T are called *strongly Morita equivalent* if there are biacts ${}_SP_T$ and ${}_TQ_S$ and morphisms $\theta : {}_S(P \otimes Q)_S \to {}_SS_S$ and $\phi : {}_T(Q \otimes P)_T \to {}_TT_T$ with certain 'nice' properties.

A right S-act A_S is *firm* if the mapping $\overline{\mu}_A : A \otimes_S S \to A, \quad a \otimes s \mapsto as$ is bijective. An act A_S over a *fair* semigroup S is called *strongly firm* if the mapping $\mu_A : A \otimes_S S' \to A, \quad a \otimes z \mapsto az$ is bijective.

In [1], fair semigroups were called *right Morita equivalent* if their categories of firm acts are equivalent. They were able to show that right fair semigroups S and T, where S' and T' have common weak right local units, are right Morita equivalent if and only if S' and T' are strongly Morita equivalent.

We call fair semigroups S and T *right Morita equivalent* if their categories of *strongly* firm acts are equivalent. Mirroring the approach of [2], we aim to show that right fair semigroups *without any additional conditions* are right Morita equivalent in this new sense if and only if S' and T' are strongly Morita equivalent. We have some promising partial results, but it is too early to tell whether this approach works without imposing any extra assumptions altogether.

This talk is based on joint research with Valdis Laan.

References

[1] V. Laan, L. Márki, Fair semigroups and Morita equivalence, *Semigroup Forum* 92, 2016, 633-644.

[2] V. Laan, L. Márki, Ü. Reimaa, Morita equivalence of semigroups revisited: firm semigroups, manuscript.

Morita invariants of semirings

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A *semiring* is an algebraic structure similar to a ring but without additive inverses. There is a zero element, but there need not be a unity. Structures analogous to modules over rings are called *semimodules*.

A semiring S has *weak local units* if for every $s \in S$, $s \in sS$ and $s \in Ss$, and *common joint weak local units* if for all $s, s' \in S$ there exist $u, v \in S$ such that $s = usv$ and $s' = us'v$. A left semimodule ${}_SM$ over S is *unitary* if $SM = M$.

A *quantale* is a complete lattice with an additional binary operation (multiplication) which is distributive over joins of any cardinality. The ideals of a semiring (defined, together with the product of two ideals, as in ring theory) form a quantale.

The concept of Morita equivalence is known classically from ring theory. It has been studied for semirings with a unit in [1]. We consider Morita equivalence on semirings with weak local units, using as basis the work of [2] on *semigroups* with weak local units.

We define *strong Morita equivalence* of two semirings S and T as the existence of a Morita context $(S, T, {}_SP_T, {}_TQ_S, \theta, \phi)$ (defined as in ring and semigroup theory), where the bisemimodules ${}_SP_T$ and ${}_TQ_S$ are unitary (from both sides), and the mappings θ and ϕ are surjective. We show that:

- two strongly Morita equivalent semigroups with weak local units have isomorphic quantales of ideals, with finitely generated ideals mapping to finitely generated ideals;
- two strongly Morita equivalent semigroups with common joint weak local units have isomorphic congruence lattices.

References

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Interaction morphisms and Turing computation

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I will introduce interaction morphisms as a means to specify how an effectful (e.g., non-deterministic, interactive I/O or stateful) computation is to be run on an abstract state machine. An interaction morphism is given by a monad $T = (T, \eta, \mu)$ and a comonad $D = (D, \varepsilon, \delta)$ on a Cartesian category with a family of maps $\psi_{X,Y} : TX \times DY \to X \times Y$ natural in X and Y and agreeing suitably with η, ε, μ, δ. Intuititively, $\psi_{X,Y}$ takes a computation and a behavior from an initial state and sends them into a return value and a final state. Interaction morphisms enjoy neat properties: they are the same as monoids in a certain monoidal category; interaction morphisms of T and D are in a bijective correspondence with carrier-preserving functors between the categories of coalgebras of D and stateful runners of T (monad morphisms from T to state monads); they are also in a bijective correspondence with monad morphisms from T to a monad induced in a certain way by D.

I will illustrate interaction morphisms on the example of Turing computation, i.e., computations interacting with a readable, writeable, walkable biinfinite tape storing symbols from a finite alphabet. Describing the monad and comonad concretely is an instructive exercise in this case.

The work on interaction morphisms continues my earlier work [2] on stateful runners and is joint with Shin-ya Katsumata (Kyoto University). Turing computation was studied in a related setting by Goncharov et al. [1].

References

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Coherence for skew near-semiring categories

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A near-semiring category is a category with two monoidal structures, $(Z, \oplus)$ and $(I, \otimes)$, where $\otimes$ distributes over Z and $\oplus$ from the right. Distributivity from the left is not required and neither is commutativity of $\oplus$. In a skew near-semiring category, the unitors, associators and distributors are just natural transformations rather than isomorphisms.

Uustalu [2] showed that skew monoidal categories satisfy a coherence theorem similar to MacLane's for monoidal categories. The free skew monoidal category Tm_X over a set of objects X has the free strict monoidal category Nf_X over X as a reflective subcategory of Tm_X. The category Nf_X is discrete and its objects are lists over X. From this it follows that there exists exactly one map between an object and its normal form.

We extend that theorem to skew near-semiring categories. Here the objects of Nf_X are given by the grammar $\mathsf{Nf}_X := Z \mid (X \otimes \mathsf{Nf}_X) \oplus \mathsf{Nf}_X \mid I \oplus \mathsf{Nf}_X$.

Rivas et al. [1] used near-semiring objects in near-semiring categories as an axiomatization of Haskell type-classes MonadPlus and Alternative. Relevant for them were categories of endofunctors with composition or Day convolution multiplicative structure and product additive structure. Switching to general functor categories skews the multiplicative structure.

This talk is based on joint work with Mauro Jaskelioff, Exequiel Rivas and Tarmo Uustalu.

References

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