

Representing regular Kleene algebras in terms of rough sets

Jouni Järvinen
Turku, Finland

This talk describes a joint work with Sándor Radeleczki [1].

A collection \mathcal{H} of nonempty subsets of U is called a *covering* of U if $\bigcup \mathcal{H} = U$. A covering \mathcal{H} is *irredundant* if $\mathcal{H} \setminus \{X\}$ is not a covering for any $X \in \mathcal{H}$. A *tolerance* is a reflexive and symmetric binary relation. Each covering \mathcal{H} induces a tolerance $R_{\mathcal{H}} = \bigcup \{X^2 \mid X \in \mathcal{H}\}$.

For any binary relation R on U , the *lower approximation* of a subset X of U is $X^{\blacktriangledown} = \{x \in U \mid R(x) \subseteq X\}$ and X 's *upper approximation* is $X^{\blacktriangle} = \{x \in U \mid R(x) \cap X \neq \emptyset\}$, where $R(x) = \{y \in U \mid x R y\}$. The set of *rough sets* is $\mathcal{RS} = \{(X^{\blacktriangledown}, X^{\blacktriangle}) \mid X \subseteq U\}$.

A *De Morgan algebra* $(L, \vee, \wedge, \sim, 0, 1)$ is a bounded distributive lattice with an operation \sim satisfying $\sim \sim x = x$ and $x \leq y$ iff $\sim y \leq \sim x$. A *Kleene algebra* is a De Morgan algebra in which $x \wedge \sim x \leq y \vee \sim y$ holds.

A double pseudocomplemented lattice $(L, \vee, \wedge, *, ^+, 0, 1)$ is called *regular* if $x^* = y^*$ and $x^+ = y^+$ imply $x = y$. If a De Morgan algebra is such that its underlying lattice is pseudocomplemented, then it forms a double pseudocomplemented lattice where $x^+ = \sim(\sim x)^*$. We say that a De Morgan algebra (or a Kleene algebra) is *regular* if its underlying lattice is a regular double pseudocomplemented lattice. Note that a De Morgan algebra defined on an algebraic lattice is always a double pseudocomplemented lattice.

It is known that if \mathcal{RS} is determined by a tolerance induced by an irredundant covering, then \mathcal{RS} forms an algebraic lattice and determines a regular Kleene algebra. We show how any regular Kleene algebra defined on an algebraic lattice is isomorphic to a rough set Kleene algebra defined by a tolerance induced by an irredundant covering.

References

- [1] J.Järvinen, S. Radeleczki: Representing regular Kleene algebras by tolerance-based rough sets, arXiv:1610.09847 (submitted to a journal).