On categorical equivalence of finite commutative rings

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The (universal) algebras **A** and **B** are said to be *categorically equivalent* if there is an equivalence functor F between varieties generated by them such that $F(\mathbf{A}) = \mathbf{B}$. The algebras **A** and **B** are said to be *term equivalent* if they have the same universe and their clones of term functions coincide. It is easy to see that term equivalent algebras are categorically equivalent.

A well known result by C. Bergman and J. Berman says that for any primes p and q and any natural number n, the Galois fields $\mathrm{GF}(p^n)$ and $\mathrm{GF}(q^n)$ are categorically equivalent [1]. We proved in [2] that in the case of finite rings semisimplicity is a categorical property. Moreover, we showed that if two finite semisimple rings are categorically equivalent then this follows from the aforenamed result of C. Bergman and J. Berman.

On the other hand, in [2] we could not find non-trivial examples of categorically equivalent finite non-semisimple rings. We proved that if two finite non-semisimple rings of prime power characteristic are categorically equivalent then their characteristics coincide.

In the present work we have focused on the commutative case. Our basic results are the following.

1. Two categorically equivalent finite commutative rings of the same prime power characteristic have isomorphic additive groups. Thus, in particular, they have the same order.

2. For every odd prime p there exist non-isomorphic but term equivalent rings of order p^3.

3. Let **R** and **S** be categorically equivalent finite commutative rings of prime characteristic p such that **R** is generated by one element over a subfield of **R**. Then **R** and **S** are isomorphic.

Problem Is it true that categorically equivalent finite commutative non-semisimple rings with the same universe are necessarily term equivalent?

References

[1] C. Bergman, J. Berman, Morita equivalence of almost primal clones I, *J. Pure Appl. Algebra* 108, 1996, 175–201.

[2] K. Kaarli, O. Košik, T. Waldhauser, On categorical equivalence of finite rings, *J. Algebra Appl* 15, 2016, 12 pp.