

On Morita equivalence of fair semigroups

Lauri Tart
University of Tartu
Tartu, Estonia

Let S be a semigroup. A right S -act A_S is called **unitary** if $AS = A$. A semigroup S is called a **right fair semigroup** (see [1]) if every subact of a unitary right S -act is unitary. Left fair semigroups are defined dually and a **fair semigroup** is one that is both left and right fair. Every fair semigroup S contains a subsemigroup S' with weak local units, called its **unitary part**.

Semigroups S and T are called **strongly Morita equivalent** if there are biacts ${}_S P_T$ and ${}_T Q_S$ and morphisms $\theta : {}_S(P \otimes Q)_S \rightarrow {}_S S_S$ and $\phi : {}_T(Q \otimes P)_T \rightarrow {}_T T_T$ with certain 'nice' properties.

A right S -act A_S is **firm** if the mapping $\bar{\mu}_A : A \otimes_S S \rightarrow A$, $a \otimes s \mapsto as$ is bijective. An act A_S over a *fair* semigroup S is called **strongly firm** if the mapping $\mu_A : A \otimes_S S' \rightarrow A$, $a \otimes z \mapsto az$ is bijective.

In [1], fair semigroups were called **right Morita equivalent** if their categories of firm acts are equivalent. They were able to show that right fair semigroups S and T , where S' and T' have common weak right local units, are right Morita equivalent if and only if S' and T' are strongly Morita equivalent.

We call fair semigroups S and T *right Morita equivalent* if their categories of *strongly* firm acts are equivalent. Mirroring the approach of [2], we aim to show that right fair semigroups *without any additional conditions* are right Morita equivalent in this new sense if and only if S' and T' are strongly Morita equivalent. We have some promising partial results, but it is too early to tell whether this approach works without imposing any extra assumptions altogether.

This talk is based on joint research with Valdis Laan.

References

- [1] V. Laan, L. Márki, Fair semigroups and Morita equivalence, *Semigroup Forum* 92, 2016, 633-644.
- [2] V. Laan, L. Márki, Ü. Reimaa, Morita equivalence of semigroups revisited: firm semigroups, manuscript.