Morita invariants of semirings

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A *semiring* is an algebraic structure similar to a ring but without additive inverses. There is a zero element, but there need not be a unity. Structures analogous to modules over rings are called *semimodules*.

A semiring S has *weak local units* if for every $s \in S$, $s \in sS$ and $s \in Ss$, and *common joint weak local units* if for all $s, s' \in S$ there exist $u, v \in S$ such that $s = usv$ and $s' = us'v$. A left semimodule ${}_SM$ over S is *unitary* if $SM = M$.

A *quantale* is a complete lattice with an additional binary operation (multiplication) which is distributive over joins of any cardinality. The ideals of a semiring (defined, together with the product of two ideals, as in ring theory) form a quantale.

The concept of Morita equivalence is known classically from ring theory. It has been studied for semirings with a unit in [1]. We consider Morita equivalence on semirings with weak local units, using as basis the work of [2] on *semigroups* with weak local units.

We define *strong Morita equivalence* of two semirings S and T as the existence of a Morita context $(S, T, {}_SP_T, {}_TQ_S, \theta, \phi)$ (defined as in ring and semigroup theory), where the bisemimodules ${}_SP_T$ and ${}_TQ_S$ are unitary (from both sides), and the mappings θ and ϕ are surjective. We show that:

- two strongly Morita equivalent semigroups with weak local units have isomorphic quantales of ideals, with finitely generated ideals mapping to finitely generated ideals;
- two strongly Morita equivalent semigroups with common joint weak local units have isomorphic congruence lattices.

References

[1] Y. Katsov, T. G. Nam, Morita equivalence and homological characterization of semirings, *Journal of Algebra and its Applications* 10(03), 2011, 445-473.

[2] V. Laan, L. Márki, Morita invariants for semigroups with local units, *Monatsh. Math.* 166(3), 2012, 441-451.