Amalgamating inverse semigroups over ample semigroups

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It was shown by Howie [1] that a semigroup amalgam $\mathcal{A} = (S; T_1, T_2)$ fails to embed if T_1 and T_2 are groups but S is not such. Generalizing this result, Rahkema and Sohail [2] showed that $\mathcal{A}$ is also non-embeddable when T_1 and T_2 are completely regular (Clifford) semigroups but S is not completely regular (Clifford). In this talk we shall consider the situation where T_1 and T_2 are inverse semigroups whereas S is a non-inverse ample semigroup. Let $\mathcal{C}$ be a class of semigroups and $T_1 \in \mathcal{C}$. We call $(S; T_1)$ an *antiamalgamation pair* for $\mathcal{C}$ if $(S; T_1, T_2)$ is non-embeddable for all $T_2 \in \mathcal{C}$. The following results will be presented.

Theorem 1. *Let S be a non-inverse ample subsemigroup of an inverse semigroup T_1. Then (S, T_1) is an antiamalgamation pair for the class of inverse semigroups.*

Theorem 2. *Let a non-inverse semigroup S be made into a right (respectively, left) ample semigroup by an inverse oversemigroup T_1 (respectively, T_2). Then $(S; T_1, T_2)$ is not embeddable.*

A subsemigroup S of an inverse semigroup T is called *rich right ample* (in T) if for all $x, y \in S$ one has $x^{-1}y \in S \cup S'$, where $S' = \{x^{-1} \in T : x \in S\}$. A *rich left ample* subsemigroup is defined dually. We call S *rich ample* if it is both rich right and rich left ample. We shall also prove the following results, concerning weak amalgamation.

Theorem 3. *Let T_1 and T_2 be inverse semigroups containing rich ample isomorphic copies of a non-inverse semigroup S. Then $(S; T_1, T_2)$ is weakly embeddable in an inverse semigroup.*

Theorem 4. *Any amalgam $(S; G_1, G_2)$, in which G_1 and G_2 are groups, is weakly embeddable in a group.*

References

[1] J. M. Howie: Embedding theorems with amalgamation for Semigroups. Proc. London Math. Soc. 12(3) (1962), 511–534

[2] K. Rahkema and N. Sohail: A note on embedding of semigroup amalgams. Acta Comment. Univ. Tartu. Math. 18(2) (2014), 261–263